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A theoretical analysis of extensive air showers III. Muon showers at large zenith angles

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Abstract. A theoretical analysis has been made of the muon component of extensive air showers incident at large angles to the vertical. Using the model described previously by de Beer *et al.* in 1966, calculations have been made of the expected numbers of muons reaching ground level as a function of threshold energy and zenith angle for primary particles of various energies.

The data have been combined with alternative primary spectra of cosmic rays, with and without an enhanced contribution from heavy nuclei, to give the expected muon density spectra. It is found that the density spectrum is sensitive to the distribution of transverse momentum among the parents of the muons and to a lesser extent to the mass composition of the primaries. There is thus the possibility of deriving information about the mean transverse momentum, and perhaps setting limits on the mass composition, from a comparison with experiment.

1. Introduction

The present paper is a continuation of a series on the theoretical analysis of extensive air shower phenomena using a conservative model for the various nuclear interaction processes in the atmosphere. The earlier results, which referred to near-vertical showers, have been published by de Beer *et al.* (1966, 1968 a, to be referred to as I and II, respectively). A preliminary report of the present work has been given by Alexander *et al.* (1968).

In the present work attention is directed at extensive air shower phenomena at large angles to the vertical, where, in view of the large thickness of atmosphere traversed, the electron component is largely absorbed and the showers consist almost entirely of muons. The need for calculations follows from the fact that some experimental data on large-angle showers are already available (Sekido *et al.* 1966, Parker 1967), and further experiments are in progress (see Rogers *et al.* 1969, the following paper). With the exception of a simple empirical treatment by Sekido *et al.* (1966) no previous theoretical analysis appears to have been made.

Interest in large-angle showers comes from the fact that the quite different view of the shower seen under these conditions, compared with the more usual near-vertical studies, might give new evidence about the two factors so intimately concerned in extensive air shower propagation: the mass composition of the cosmic ray primaries and the character of the high-energy nucleon–air-nucleus interaction.

An example of an interaction parameter about which large-angle extensive air shower studies (and indeed muon studies at all angles) should give information is the mean transverse momentum $\langle p_T \rangle$ of the pions whose decay is presumed to be responsible for the detected muons. There is great contemporary interest in this parameter in view of the suggestion from studies of the muon component of near-vertical showers, by Earnshaw *et al.* (1968), that the mean value may be significantly higher than the 0·4 GeV/*c* expected from more direct measurements (see also the work of de Beer (*et al.* 1968 b) for a discussion of the derivation of $\langle p_T \rangle$ from muon lateral distribution measurements). The possibility of determining $\langle p_T \rangle$ follows from the fact that the width of the muon lateral distribution is largely determined by this quantity, as is the case for near-vertical showers, but in the large-angle showers there is greater sensitivity and theoretical treatments are potentially

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more accurate because the region of height in the atmosphere from which the muons come is relatively better defined owing to the energy of the parent pions being higher.

The sensitivity to the mass composition of the primaries arises because of the opposite effect on muon and electron numbers of increasing the atomic weight of the primaries, the muon number increasing and the electron number decreasing as A increases. Thus, by using the measured electron size spectrum of near-vertical showers as a datum and working up through the atmosphere to the primary spectrum and back to the muon density spectrum at large zenith angles, it is possible, in principle, to determine the mean value of A for the primaries by comparison with experiment.

In what follows calculations are made of the muon energy spectra and lateral distributions for primaries of unique energy and this is followed by derivations of the expected muon density spectra for two alternative primary spectra.

2. Muon characteristics for primary particles of unique energy

2.1. *Details of the model and method of calculation*

The adopted model for the high-energy interaction process is that described in I and referred to there as model II. Briefly, it assumes that only pions are important as the parents of muons, that the pion multiplicity is proportional to $E_p^{1/4}$ and that their energy spectrum follows the empirical relation given by Cocconi *et al.* (1961), a relation that is found to be reasonably accurate at machine energies and is applicable to primary energies as high as 10^{14} ev (see the discussion by Craig *et al.* 1968).

For most of the calculations it is assumed that the distribution of transverse momentum p_T among the secondary pions is given by the relation

$$N(p_T)\, dp_T = \frac{p_T}{p_0^2} \exp\left(-\frac{p_T}{p_0}\right) dp_T$$

where the mean value of p_T $(= 2p_0)$ is taken as 0·4 GeV/c. In fact, later work has shown that this relation is not valid at low values of p_T but the discrepancy does not affect the present analysis.

Fluctuations in inelasticity and multiplicity have not been included so far. Presumably this neglect is not important when the results are applied to the production of large numbers of muons but significant errors might be expected at low multiplicities. In most of what follows the produced (as distinct from detected) multiplicities are comparatively large.

The method of calculation is similar to that described in I, viz. the use of a lattice giving sea-level muon characteristics expected for nucleon and pion interactions transferring various amounts of energy at a variety of levels in the atmosphere. Primary particles are then followed down through the atmosphere and the resulting effects from the various interactions are summed.

Compared with the case for near-vertical showers two effects have enhanced importance at large zenith angles: the μ–e decay process and the geomagnetic and scattering deflection of the surviving muons in their passage through the atmosphere. Both effects are important because of the much greater muon path lengths involved.

In view of the dependence of the geomagnetic deflection on the orientation and location of practical detectors, the calculation of the lateral distribution of muons was divided into two parts: in the first, the effects of the transverse momentum distribution and Coulomb scattering were taken into consideration and, in the second, geomagnetic deflection alone was considered.

2.2. *Total number of muons and energy spectrum*

The variation of the mean number of muons $\bar{N}_\mu$ with primary proton energy E_p is given in figure 1, the data referring to three muon threshold energies and zenith angles of 60°, 75° and 84°. The muon number is also given for vertical showers for comparison. The reduction in total number of muons with increasing angle, which arises in the main from the loss of slow muons by μ–e decay, is very marked. Comparing $\theta = 75°$ and 84° for a muon threshold of 100 GeV it is seen that there are more muons at the larger angle; this

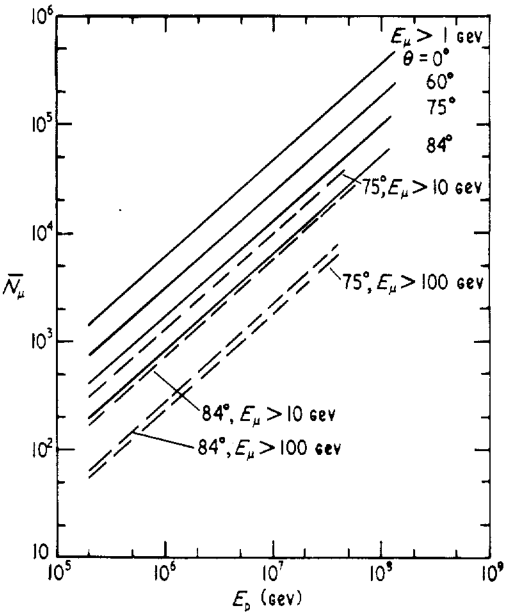

Figure 1. Mean number of muons at sea level as a function of primary proton energy for three muon energy thresholds and two zenith angles. The results for $\theta = 0°, E_\mu > 1$ GeV, are also shown for comparison. (Note, this last result differs somewhat from that given by de Beer *et al.* (1966). The present result is more accurate.)

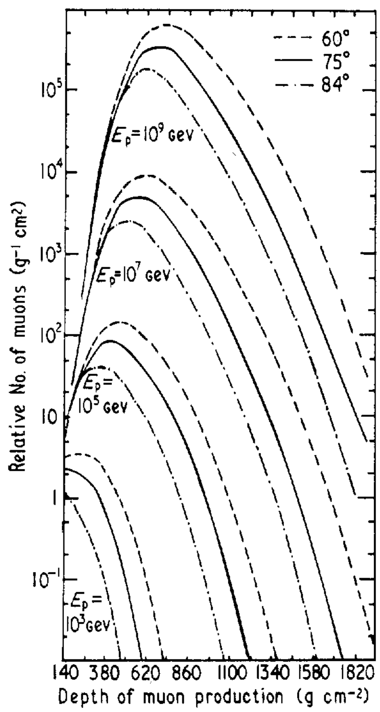

Figure 2. Incremental production of muons in the atmosphere for three zenith angles. The depth is measured along the length of the path of the shower. The results refer to primary protons and $E_\mu > 1$ GeV.

arises from the well-known enhancement of the inclined high-energy muon flux owing to the parent pions being produced in regions of the atmosphere where the density is progressively lower as the angle increases. (It has recently been suggested, by Bergeson *et al.* (1967), that this enhancement may be largely suppressed at high energies, $E_{\mu} > 1000$ GeV; however, such energies contribute little to the present analysis and there is strong evidence, e.g. Nash and Wolfendale (1968), that at lower energies the enhancement factor is present.)

2.3. *Incremental production in the atmosphere*

The geomagnetic and scattering displacement depend on the heights of origin of the muons, and these can be seen by reference to figure 2 which shows the incremental production of the muons as a function of atmospheric depth, measured along the length of the path. The observed features can be understood in terms of the factors mentioned in § 2.2.

2.4. *Lateral distribution of muons*

The muon density has been calculated as a function of lateral distance for primary protons incident at 60°, 75° and 84° and the lateral distributions for two primary energies are shown in figure 3. Here the radial distance is measured perpendicular to the shower axis

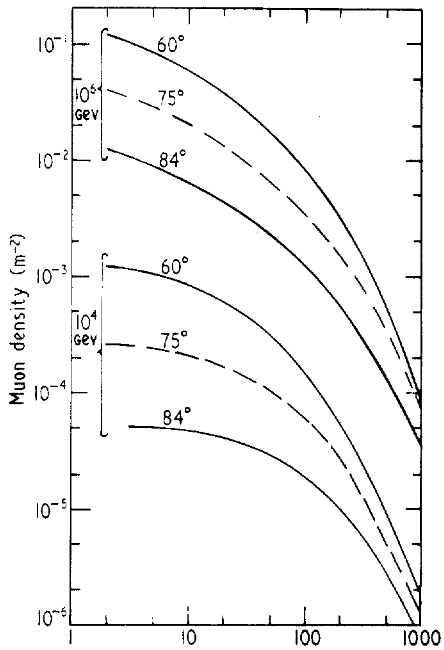

Figure 3. Lateral distribution of muons with $E_{\mu} > 1$ GeV at sea level produced by primary protons for two primary energies and three zenith angles (mean transverse momentum = 0·4 GeV/c).

and the calculations refer to a mean transverse momentum of 0·4 GeV/c. The transformation to some other value of $\langle p_{\mathrm{T}} \rangle$ is straightforward: if the new value is 0·4f GeV/c then each density must be divided by f^2 and transferred to a new radius equal to f times the original. The lateral distributions shown in figure 3 include the effect of Coulomb scattering in the atmosphere but not magnetic deflection, which depends on the orientation of the shower trajectory with respect to the geomagnetic field.

The width of the muon lateral distribution is found to be almost independent of the primary energy, a feature that follows from the fact that the decrease in height of the muon production level (figure 2) with increasing primary energy is largely compensated by the decrease in mean energy of the muons produced.

2.5. *Sensitivity of muon number to mass of primary particles*

Assuming, as in previous papers (I and II), that a shower initiated by a heavy primary of mass A and energy E can be regarded as the superposition of A proton-initiated showers, each of energy E/A, the number of muons in heavy-nucleus-initiated showers can be obtained. Inspection of figure 1 shows that $N_{\mu} \propto E^{\alpha}$ with $\alpha = 0{\cdot}88$ for all values of θ so that $N_{\mu}(A) \propto A^{1-\alpha}$, i.e. $N_{\mu} \propto A^{0{\cdot}12}$; thus, there is an increase in the number of muons by about 1·5 in going from primary protons to heavy nuclei with $A = 30$.

The assumption of a superposition of proton showers neglects the case where several of the nucleons in the heavy nucleus interact with their opposite numbers in the air nuclei. These collisions will be important when considering sea-level muons of high energy and the magnitude of the muon density at large distances from the shower axis, say $r > 500$ m at $\theta = 60°$. However, in the present case interest centres on the density spectrum and this is conditioned mainly by the density comparatively close to the axis, in the region of $r = 100$ metres, where the effect of these multiple collisions will be small. Furthermore, it is relevant to point out that the important region of r is too great for the results to be seriously affected by the inaccuracy in the form of $N(p_{\mathrm{T}})$ at low p_{T} (see § 2.1).

3. The primary cosmic ray spectrum

The rapidity with which the intensity of primary cosmic rays falls off with energy limits direct measurement of the energy spectrum to below 10^{14} ev. The only evidence on which estimates of the intensity at higher energies may be based comes from extensive air shower observations, and, of the data available, the size spectrum (i.e. the frequency as a function of the total number of electrons) is probably the most useful.

In the present work the model of I has been used to relate the number of electrons at sea level to the energy of the incident primary particle for various assumed nuclear masses. The adopted sea-level size spectrum has been taken from a survey of all available measurements and covers the range 10^4 to 10^8 particles. As mentioned earlier, the conversion to a primary spectrum has been made for alternative assumptions as to the mass composition of the primaries at energies above 10^{15} ev where there is strong evidence for a change in slope of the spectrum. Below 10^{15} ev a common primary spectrum has been taken, this being derived from extensive air shower data which correspond to energies as low as 5×10^{13} ev. It is assumed that the mass composition is independent of energy in this region, that is the usual relative intensities of p, α, light, medium, heavy and very heavy nuclei have been taken (specifically, figures given by Ginzburg and Syrovatsky (1964)). The form of the spectrum is $N(>E) \propto E^{-1{\cdot}6}$, a relation which agrees with the suggestions of several other workers. Some confidence in it follows from the fact that extrapolation to lower energies passes through the direct measurements in the energy region 10^9–10^{10} ev. Further confirmation of this procedure comes from the work of Malholtra *et al.* (1966) whose direct measurements extend to about 6×10^{14} ev, although the uncertainties on the absolute intensities in that work are rather great.

Turning to energies above 10^{15} ev we follow the procedure indicated earlier, and also adopted in II, that is to use the size spectrum at sea level together with alternative mass compositions: protons alone and a composition which might obtain if there were galactic modulation of the mass spectrum present at lower energies.

The need for considering protons alone as a possibility at energies above 10^{15} ev arises from a number of considerations:

(i) There is evidence that at energies in the region of 10^{17} ev the primaries may be predominantly protons (Linsley and Scarsi 1962).

(ii) There are attractive origin theories in which the heavy nuclei are fragmented by the intense radiation fields surrounding the sources (Kuzmin and Zatsepin 1968).

(iii) Earlier measurements which appeared to indicate that the majority of primaries in the range 10^{15}–10^{18} ev are heavy nuclei, such as studies of fluctuations and the frequency of multiple shower cores, have been shown to be inconclusive (Adcock *et al.* 1968, Bohm *et al.* 1968).

The derived primary proton spectrum is shown in figure 4 where it is designated by spectrum A.

The alternative spectrum, referring to galactic modulation, is more difficult to derive in view of the variety of forms that the modulation could take. In II an exponential rigidity modulation was adopted but we now feel that this is unlikely and instead the form suggested by Ginzburg and Syrovatsky (1964) is used. Following these workers, an increase in exponent of 0·5 has been imposed on the energy spectrum of each mass component above a

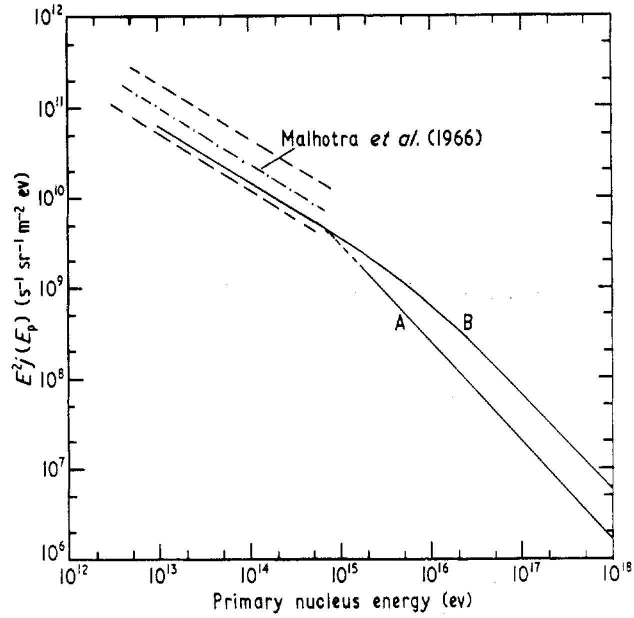

Figure 4. Alternative primary spectra. These give the measured sea-level electron size spectrum at sea level using the calculations described in papers I and II. The direct measurements of Malholtra *et al.* (1965) are shown for comparison.

constant rigidity. The value of the rigidity at which the change of slope occurs has been taken as the sole variable and its position has been determined for the condition that the ensuing size spectrum, summed over the contributions from the various primary masses, should be nearest to the result of our survey of the size spectrum measurement. For a value corresponding to $1{\cdot}5 \times 10^{15}$ ev for protons the fit is singularly good. The modulated spectrum, expressed in terms of energy per nucleus, is shown in figure 4, denoted by spectrum B.

It is relevant to point out that the exponential modulation used in II gives a much poorer fit to the size spectrum in addition to corresponding to a modulation mechanism that is hard to reconcile with current ideas about galactic magnetic fields.

An important point to be stressed concerns the role of fluctuations in shower size. It is well known (e.g. Peters 1961, Bray *et al.* 1965) that if fluctuations are neglected a sharp cut-off in rigidity will give rise to the observed size spectrum. However, in our view (see, for example, the discussion in I) fluctuations in size are large, particularly for protons, and with a sharp cut-off in rigidity this would give a resultant size spectrum that is too steep, because the contribution to the size spectrum becomes progressively less with increasing primary mass.

Of interest is the variation of mass composition with primary energy and measured size and an indication of this is given in figure 5 where, as a measure of composition, the fractions of primaries which are protons are given for the two primary spectra considered. The fraction relevant to a particular experiment would depend on the type of triggering adopted; for all-particle triggering the fraction would approximate to that plotted against size in the figure. For muon triggering the fraction would be lower because of the increased efficiency of muon production for heavy primaries.

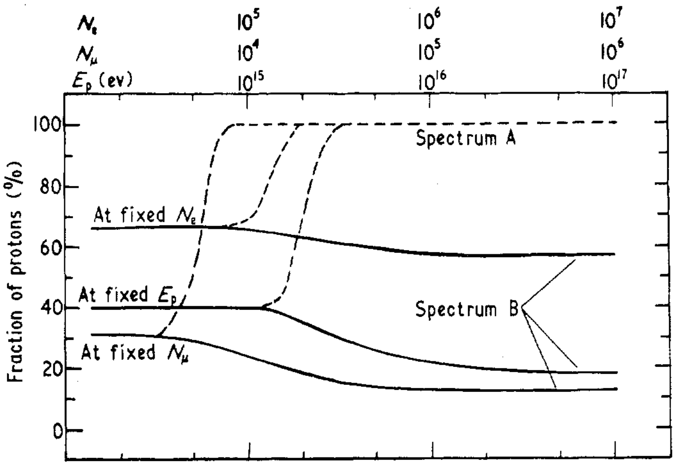

Figure 5. Fractions of primary cosmic rays which are protons for the alternative spectra A and B expressed as a function of primary energy per nucleus, sea-level shower size and sea-level muon number. The form of the transition in the region of 10^{15} ev for A is rather arbitrary.

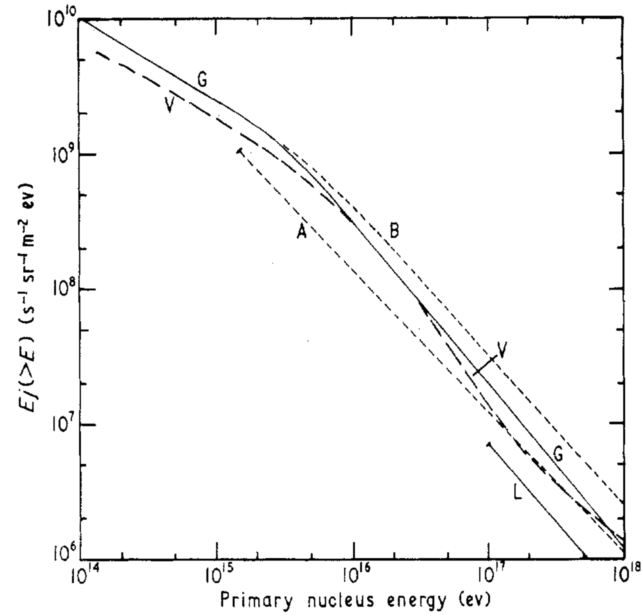

Figure 6. Comparison of the spectra quoted by various authors. G, Greisen 1965; V, Vernov and Khristiansen 1968; L, Linsley 1965; A and B, present work.

Finally, a comparison is made in figure 6 of a number of recent estimates of the primary spectrum. The divergence between these and the assumed spectra A and B is due to several causes, amongst them the different compositions assumed. The other estimates contain the implicit assumption that the composition remains unchanged over the energy range considered. Calculations using the present model and a mass composition constant over all energies result in a primary spectrum very close to that of Greisen: with a modulated composition the intensities are higher, and, with protons only, the intensities are lower.

4. The density spectrum of muons

4.1. *Form of the density spectrum*

The lateral distributions have been folded in with the primary spectra to give the expected muon lateral distributions for various angles. The details of the calculation are as follows. Using the lateral distributions of the type given in figure 3 the radial distance from the shower axis, $r(\Delta)$, has been found as a function of primary energy for a variety of values of density Δ. The integral density spectrum then follows immediately as

$$N(>\Delta) = \int_{E_{\min}}^{\infty} \pi r^2(\Delta) j(E_{\mathrm{p}})\, \mathrm{d}E_{\mathrm{p}}$$

where $j(E_{\mathrm{p}})$ is the differential primary intensity.

$E_{\min}$ is given by the upper of two limits:

(i) The energy below which the primary particle cannot produce the number of muons under consideration (this number is taken as 2). This limit is only important at very low densities.

(ii) The energy corresponding to a minimum radial distance $r_{\min}$ for the density value in question. This radial limit is again set by instrumental considerations and concerns the requirement that the density variation across the detector should not be great. A value $r_{\min} = 3$ metres has been adopted here (see following paper by Rogers *et al.* 1969). Neither limit plays an important part in the calculations.

The derived density spectra are shown in figure 7 for the case where the mean transverse momentum is 0·4 GeV/c and geomagnetic deflection is neglected.

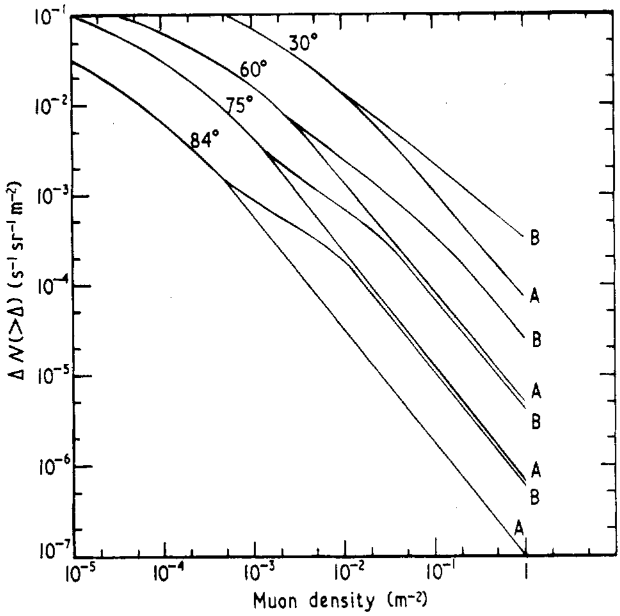

Figure 7. The density spectra of muons with energy above 1 GeV for four zenith angles and alternative primary spectra ($\langle p_{\mathrm{T}} \rangle = 0{\cdot}4$ GeV/c). Curves A and B are derived from the primary spectra A and B respectively. Geomagnetic effects have been ignored and the spectra therefore refer to showers incident along the field lines. For other directions the absolute values will change but the relative B-to-A values will be virtually unaltered.

It can be seen that the point at which the relatively large difference between the density spectra for alternative primary spectra appears moves to higher densities as the zenith angle decreases. This is because the point corresponds to the primary energy at which the alternative primary spectra start to diverge (figure 4).

4.2. *Sensitivity to transverse momentum*

The transformation of lateral distribution from one value of $\langle p_T \rangle$ to another was discussed in § 2.4. The corresponding density spectra for $\theta = 60°$ and $\langle p_T \rangle = 0{\cdot}4$, $0{\cdot}6$ and $0{\cdot}8$ GeV/c are shown in figure 8, where it is seen that there is considerable sensitivity

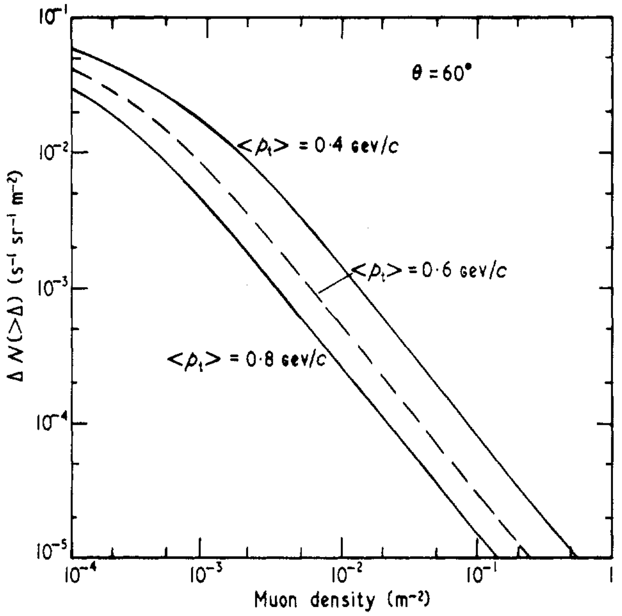

Figure 8. Sensitivity of density spectrum to $\langle p_T \rangle$ for $\theta = 60°$, primary protons and $E_\mu > 1$ GeV.

to the value of $\langle p_T \rangle$. In so far as $\langle p_T \rangle$ most probably increases with interaction energy, the corresponding density spectra would be slightly steeper than those shown, cutting across from the lower to higher $\langle p_T \rangle$ spectra as the density, and thus the mean interaction energy, increases.

4.3. *Effect of geomagnetic deflection*

The calculations described so far have not included the effect of the Earth's magnetic field and they are therefore only directly applicable in practice to that small fraction of showers incident along the Earth's field lines.

An accurate treatment of geomagnetic displacements in the general case is difficult because of the distribution of both the heights of origin of the muons and their momenta. However, in the experiment with which the main comparison will be made (Rogers *et al.* 1969) the bulk of the data come from zenith-angle regions where the geomagnetic effects are small and thus only approximate calculations are necessary. The order of magnitude of the magnetic displacement in an actual experiment can be seen with reference to the apparatus of Rogers *et al.* (1969). For the appropriate latitude and orientation of the array, the calculated mean magnetic displacements are 220, 950 and 3200 m for zenith angles of 60°, 75° and 84° respectively and these are to be compared with mean radial distances for transverse momentum and scattering alone of 380, 530 and 710 m. It is clear therefore that at angles above about 70° the bulk of the particle displacement is due to magnetic deflection.

Two methods of calculating the influence of the Earth's field have been adopted. In the first, the root-mean-square displacement from geomagnetic deflection alone was calculated and this was combined with the same quantity for the distribution neglecting magnetic deflection by adding in quadrature, and the distribution in displacement for this new r.m.s. value was calculated assuming no change in shape. This first-order calculation neglects the elliptical character of the shower but imposes some sort of mean shape on it.

In the second method all muons at a given distance from the axis were assumed to suffer a geomagnetic displacement in the appropriate direction proportional to the radial distance and the resulting elliptical lateral distribution was used to give the density distribution to be used in the comparison with experiment. This assumption is reasonable because over the important middle radial distance region of a shower the mean energy is roughly inversely proportional to the radial distance.

The results of the two methods were close out to 70° and this lends support to the validity of the calculations for zenith angles as far as this value. Finally, the fact that the observed rate of multiple muons against angle out to 80° has the same shape as that calculated using the estimated geomagnetic correction in the case of the experiment of Parker (1967, see Rogers *et al.* 1969, figure 6) gives further confidence in the calculations.

5. Discussion

The results presented in figures 7 and 8 show the sensitivity of the density spectrum of muons in inclined directions to the mass composition of the primaries and to the mean values of the transverse momentum for the adopted model of high-energy interactions. The sensitivity is such that, if it were know that the adopted model was correct in every particular (but with unknown $\langle p_T \rangle$) and if accurate experimental data were available, then for primary energies below 10^{15} ev (sufficiently low densities where there is no difference between A and B, figure 7) it would be possible to determine $\langle p_T \rangle$ uniquely. If it is now assumed that $\langle p_T \rangle$ changes only slowly with interaction energy there is the possibility of distinguishing between A and B by comparison of experiment and theory at the higher sea-level muon densities.

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